THERMOCAPILLARY CONVECTION IN THIN FLUID LAYERS

L.G. Badratinova
Lavrentyev Institute of Hydrodynamics
Novosibirsk 630090, USSR

The paper deals with consideration of a long-wave approximation for the problem of thermocapillary convection in two-layer fluid, confined between parallel solid plates, with a constant temperature gradient sustained between the plates. The allowance is made of the thermocapillary, gravitational and capillary waves at the interface. The problem of convection description is reduced to a system of two quasilinear equations to determine the interface level $h (0 < h < 1)$ and pressure p of one (upper) medium. The conservation laws of masses of fluid layers are expressed by these equations in the form

$$h_t + \operatorname{div}(a_1 \nabla \Delta h + a_2 \nabla h + a_3 \nabla p) = 0$$
$$\operatorname{div}(a_4 \nabla p) = \operatorname{div}(a_5 \nabla \Delta h + a_6 \nabla h) \tag{1}$$

Here ∇ , Δ , div are the two-dimensional differential operators in the "longitudinal" coordinates x, y. The coefficients a_i depend on the thickness $h(x, y, t)$ and five dimensionless parameters of the problem. Among these parameters we take the ratios of media heat conductivity coefficients, $\varkappa_*$, the ratios of dynamic viscosity coefficients, μ_*, and three similarity criteria characterizing an influence of thermocapillary, gravitational and capillary forces. In equations (1) the function $a_4 > 0$, $a_1 \geqslant 0$, the function a_1 being equal to 0, when $h=0$ or the capillary forces do not work.

The question concerning the solvability of the problem with the initial data

$$h = h_o(x, y) \quad \text{when } t = 0 \qquad (0 < h_o < 1) \tag{2}$$

for system (1) is quite complicated in the general case. For the periodic function h_o, that belongs to the Hölder class $C^{4+\alpha}$, $\alpha \in (0, 1)$, the local solvability of problem (1), (2) can be proved for a class of indefinitely differential periodic functions. If the capillary forces are neglected, the summands, proportional to $\nabla\Delta$ h, disappear in (1). In this case there is an analogy between equations (1) and the Backley - Leverette equations in the theory of filtration of immissible fluids, for which the Cauchy problem is studied well.

The paper studies the stationary solutions of equations (1) for periodic and biperiodic flows, solitons with an immovable interface such as "convex", "concave" or "bore", one-dimensional and axisymmetric flows with an immovable interface crossing the solid plates. It is shown that system (1) has no one-dimensional periodic solutions in the form of stationary travelling waves. A limiting case for system (1), (2) calls special attention, in which the ratios of densities ρ_* and dynamic viscosity coefficients μ_* vanish, and the ratio of heat conductivity coefficients $\varkappa_*$ tends to a finite limit. The problem is reduced to a closed equation that describes evolution of thickness for a two-layer "fluid-gas" system under a cap. In this case we consider an analogy between this problem and the problem of thermocapillary convection in a fluid layer with a free boundary, for which the Newton heat exchange condition is fulfilled.

A review of the other studies concerning the thermocapillary convection in thin fluid layers, performed in the Lavrentyev Institute of Hydrodynamics, is given. An interesting case is that of anomalous dependence of the surface tension σ on temperature, i.e. $\sigma = \sigma_0 + \eta(T-T_0)^2$, where σ_0, $\eta > 0$ are the constants, T_0 is the temperature of the equilibrium free surface. In this case an increase in disturbances, periodic and nonperiodic with respect to a spatial coordinate, was discovered for the finite time.